the curvature, K_1, is found to be

$$K_1 = -\frac{a^{2/3}b^{2/3}(\alpha - \beta)}{3\beta^{2/3}\alpha^{2/3}(b^{4/3}\alpha^{2/3} + a^{4/3}\beta^{2/3})^{3/2}},$$

and this becomes infinite only when $\alpha = 0$ or $\beta = 0$. Thus cusps occur on the evolute at the points $\left(\pm\frac{c^2}{a}, 0\right)$, $\left(0, \pm\frac{c^2}{b}\right)$ corresponding to points of maximum and minimum curvature on the ellipse.

RECENT PUBLICATIONS.

REVIEWS.

Exercises in Algebra (including trigonometry). By T. P. NUNN. 2 volumes. London and New York, Longmans, 1913–1914. Vol. I, 11 + 421 pp. Vol. II, 11 + 551 pp. Price 4 + $6\frac{1}{2}$ shillings.

The Teaching of Algebra (including trigonometry). By T. P. NUNN. London and New York, Longmans, 1914. 12mo, 14 + 616 pp. Price $7\frac{1}{2}$ shillings.

Among the contributions that have recently been made to the solution of the problem of teaching secondary and collegiate mathematics, Dr. Nunn's work is one of the most significant. His plan is to build up a course which shall combine the most vital and essential parts of algebra, trigonometry, analytic geometry, and elementary calculus, and present them in a way which shall meet the needs both of the student who is studying mathematics merely for its cultural, disciplinary, or informational value, and of the student who plans to go on to further study of the subject. Many teachers have serious doubts as to the wisdom of giving the same work to these two different classes of students; but Dr. Nunn's course seems to the reviewer to furnish a successful demonstration that the thing can be done.

The two volumes of *Exercises* are designed for the student, and the third volume, on *Teaching,* indicates to the teacher the reasons for the order in which topics are taken up, and gives suggestions as to methods of presentation. The first volume of the *Exercises* contains little detailed explanation for the student, this being left for the teacher to give verbally on lines indicated clearly in the *Teaching.* The second volume of the *Exercises,* however, contains more discussion of the topics treated, with the object of developing in the student the ability to read and understand mathematical writing.

The work in elementary algebra is centered about two topics, the graph and the formula. By making skillful and persistent use of both, the author gives a vitality and meaning to the fundamental laws of operation which the student can rarely gain through the mere study of the axioms and formal laws. To give one illustration, the distributive law of multiplication is not first *stated,* and then

illustrated by geometrical figures, as is done in most texts, but rather it is *developed* in the mind of the student as a convenient short cut in area problems that require the value of $ac \pm bc$. Thus the usual order of procedure is here reversed, factoring being considered *before* multiplication as a formal algebraic process. Furthermore, the formal laws are first used with signless numbers, and not until the second section of the course (27th exercise, page 155) are directed numbers introduced. Much is made of approximation formulas as illustrations of the usefulness of such identities as $(a \pm b)^2 = a^2 \pm 2ab + b^2$.

Direct proportion is introduced early, as a particularly important relation between variables, and the opportunity is taken to define the trigonometric functions of acute angles. This is surely an admirable idea, and offers a suggestion that might well be considered by American writers of texts on elementary algebra. After the introduction of directed numbers, arithmetic series are studied, and the summation applied to the problem of finding certain areas in a manner that prepares the ground very skillfully for the elementary work in integral calculus which is to follow not much later. After an elementary study of the quadratic equation and the principles of transformation of coördinates by translation, the remarkable innovation is introduced of taking up "area functions" (Exercise 47, p. 250). The method employed is the development of what the author calls "Wallis's Law," and may be illustrated as follows: in order to obtain the area under the curve $y = kx^2$, from $x = 0$ to $x = x$, the student is led to discover by induction that the ratio of the required area to that of the rectangle of base x and height kx^2 is approximately

$$\frac{0^2 + 1^2 + 2^2 + \cdots + m^2}{(m+1)m^2} = \frac{2m+1}{6m} = \frac{1}{3} + \frac{1}{6m},$$

where m is the number of subdivisions of the base. The conclusion is, that we may with no appreciable error take $\frac{1}{3}kx^3$ as the area under the curve. Similar work is carried through for other areas. At about this same period the subject of gradients and simple differentiation formulas are also introduced. It certainly demands great skill on the part of the teacher to present this work to boys of 15 and 16 years, and it would be interesting to know if the experiment has been tried in any American secondary schools.

The subject of logarithms is introduced by means of "growth problems," that is, problems depending upon the value of a^x for certain integral or fractional values of x. The historical method of Napier in constructing his system of logarithms is then employed, a procedure that seems rather questionable, but which is certainly worthy of careful experimental comparison with the usual method of approach.

Part II of the *Exercises* carries the student much farther into the field of what we know as collegiate mathematics, including functions of two variables, the trigonometry of the sphere, complex numbers, periodic functions (this section containing also a very good geometric introduction to the hyperbolic functions), and limits, which includes linear differential equations, Taylor's Theorem, and

even a good presentation of Weierstrass's derivativeless continuous function, of Peano's space-filling curve, and of Moore's "crinkly curve."

Constant use is made of the applications of mathematics. Thus, in Part I the introductory topics, the graph and the formula, are employed in connection with problems from mensuration, commercial arithmetic, physics, engineering, botany, and astronomy. Navigation is begun as a simple illustration of the use of the trigonometric functions (problems in parallel sailing, pages 118, 119) and is continued and elaborated in Part II, pages 105–142. In the latter place we find Sanson's, Lambert's, and Mercator's projections explained and applied in numerous problems; further, great circle sailing, the construction and use of gnomonic projections, and the elementary principles of practical astronomy (pages 143–169) are considered. The increasing importance of navigation in our national life suggests the advisability of enriching our mathematics courses, both in secondary school and in college, with much of this material. The subject of annuities and life insurance occupies pages 63–78 of Part II, compound interest having been considered briefly in Part I as an application of geometric progressions. The usually under-valued subject of statistics is given a very satisfactory treatment at the end of Part II (pages 431–514).

The mere mention of these topics is sufficient to show that the author has laid out an ambitious program for the secondary school course; but the original and stimulating manner in which all of the topics are presented tends to remove one's doubts as to the feasibility of carrying the young student into so much of the work which we have usually placed much later in the mathematics course. At any rate, we have in Dr. Nunn's work a valuable contribution to a most important problem, and one that can not fail to be suggestive and stimulating to any teacher of mathematics. The three volumes should find a place in every high school and college library.

R. B. McClenon.

Interpolation Tables, or Multiplication Tables of Decimal Fractions, giving the product to the nearest unit of all numbers from 1 to 100 by 0.01 to 0.99 and from 1 to 1,000 by 0.001 to 0.999. By Henry B. Hedrick. Carnegie Institution of Washington. Washington, 1918. 9 + 139 pp. Folio. Price $5.00.

Until about the beginning of the eighteenth century the multiplication of one number by another must have been considered a serious undertaking. In the old arithmetics very simple examples of the processes are given with as painful detail as is now given to an example in the extraction of the cube root. Napier's "bones" were hailed as a wonderful invention and yet they are of little use to one who knows his multiplication table as far as the nines. As late as 1841 an immense and complicated table was published by Bretschneider giving the product of all numbers up to 100,000 by 2, 3, 4, 5, 6, 7, 8 and 9. Nevertheless, as early as 1610 a table giving the product of any two numbers less than 1,000 was published by Herwart von Hohenburg in an immense folio volume of over a thousand pages.